Fuzzy Logic

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Presentation Outline

Introduction to Fuzzy logic

- Fuzzy logic is an extension of multivalued logic
- Natural language rules
- Aristotle, later Lofti A. Zadeh in 1965 and 1973
- Japan, later on west

Figure 1: Lofti A. Zadeh

Fuzzy logic vs Crisp logic

Example

Carmen is 18 years old. Is she old?

Crisp[1] **true**/**false**

Fuzzy **true**, **false** or the **degree** of *oldness*

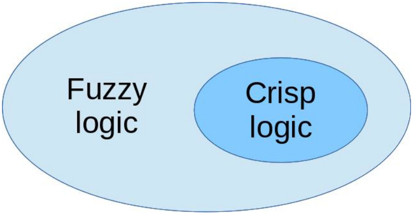


Figure 2: The classical set theory is a subset of the theory of fuzzy sets

[1] In this context referred also as a *Boolean* or *bivalent* logic

Crisp Set

Theory of Sets (formerly Classes) was conceptualized by George Cantor in 1870's.

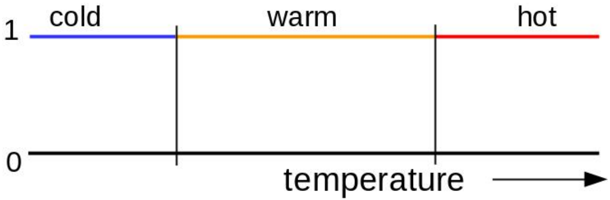


Figure 3: Crisp set illustration. The element either is fully member of a set or is not a member at all.

Sorites Paradox

When does a heap of grains stops being heap, if we are removing one grain at a time?

Figure 4: At what point exactly does blue becomes red? Sorites paradox [4].

$$\boldsymbol{Bald(0)}$$

$$\boldsymbol{Bald(n) \rightarrow Bald(n+1)}$$

$$\boldsymbol{\therefore Bald(10000)}$$

Fuzzy Sets

In mathematics, fuzzy sets are sets whose elements have *degrees of membership*, described by a *membership function* [1].

- Degree of membership is defined in interval[2] $[\mathbf{0}, \mathbf{1}]$
- Elements can have different degree of membership to different fuzzy sets
- If the uncertainty is not handled, we talk about **type-1** fuzzy sets, **type-2** otherwise

[2] In theory, it could be higher than 1, but in practice it is almost never used

Fuzzy Set Interpretation

How do we represent *numerical* value in a fuzzy set? With the use of *linguistic variables* [2], **not** probabilities.

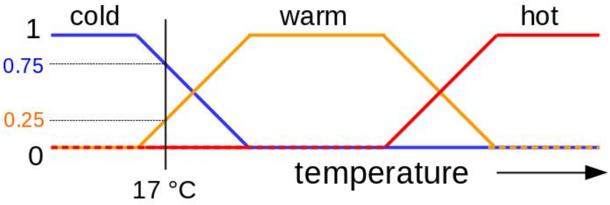


Figure 5: Example interpretation of fuzzy sets. At the given temperature point, we can tell that the measured medium is "not hot", "slightly warm" and "almost cold". It does not mean that the chance the water is cold is 75%.

Formal Definitions

Definition

Let U be the *universe of discourse* and x be the element in it. The *membership function* f^A assigning *degree of membership* μ_A:

$$f^A(x) : \in U \to \mu_A(x) \in [0, 1]$$

Definition

A fuzzy set A is expressed as a set of ordered pairs (tuples), given that $\mu_A(x)$ is a degree, to which x a member of A:

$$A = \{(x, \mu_A(x)) \,|\, x \in U\}$$

Standard Fuzzy Set Operations

Given that $A, B \in U$ and u is an element in universe U:

Complement $\mu_A(u) = 1 - \mu_A(u)$

Intersection $\mu_{A \cap B}(u) = min\{\mu_A(u), \mu_B(u)\}$

Union $\mu_{A \cup B}(u) = max\{\mu_A(u), \mu_B(u)\}$

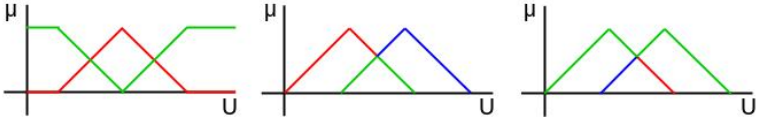


Figure 6: The complement $\mu_{\bar{\mathbf{A}}}$, the intersection $\mu_{\mathbf{A} \cap \mathbf{B}}$ and the union $\mu_{\mathbf{A} \cup \mathbf{B}}$ (green).

Fuzzy Set Operations Truth Tables

Table 1: The truth tables for **AND**, **OR** and **NOT** operations

A	**B**	**min(A,B)**
0	0	0
0	1	0
1	0	0
1	1	1

A	**B**	**max(A,B)**
0	0	0
0	1	1
1	0	1
1	1	1

A	**1-A**
1	0
0	1

It is no coincidence, that these truth tables for binary fuzzy sets are identical to their Boolean counterparts[3].

[3]DeMorgan's law, associativity, comutativity and distributivity also apply.

Triangular Norm (T-norm)

A T-norm is a **continuous** function $T : [0, 1] \times [0, 1] \to [0, 1]$, satisfying these axioms:

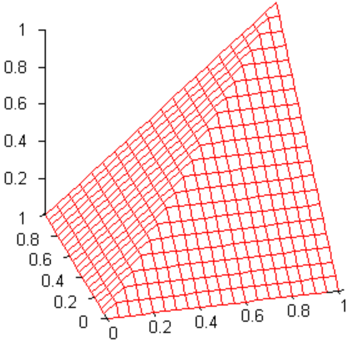
Neutrality[4]	$T(a, 1) = a$
Commutativity	$T(a, b) = T(b, a)$
Monotonicity	$T(a, b) \leq T(c, d)\ if\ a \leq c\ and\ b \leq d$
Associativity	$T(a, T(b, c)) = T(T(a, b), c)$

T-norm is used to customize the fuzzy **intersection** (conjunction). The fuzzy **union** (disjunction) uses the S-norm (or T-conorm).

[4] Also referred to as a *boundary condition*.

The Most Common T-norms

$$\mathbf{T}_{\mathbf{min}}(a, b) = min\{a, b\}$$



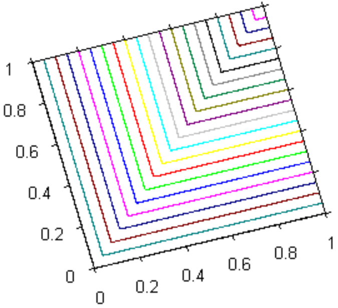
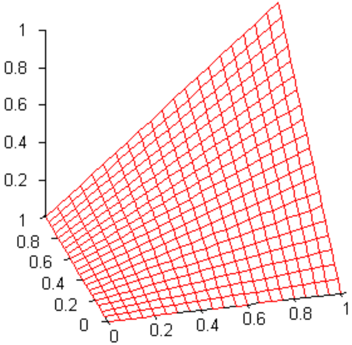


Figure 7: **Minimum** (Gödel) T-norm is the most common one

The Most Common T-norms

$$\mathbf{T}_{\mathbf{prod}}(a, b) = a \cdot b$$



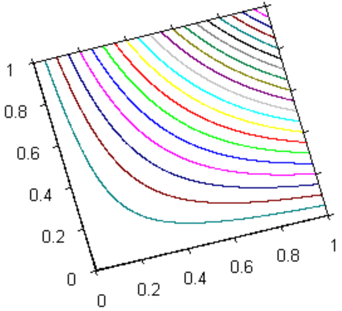
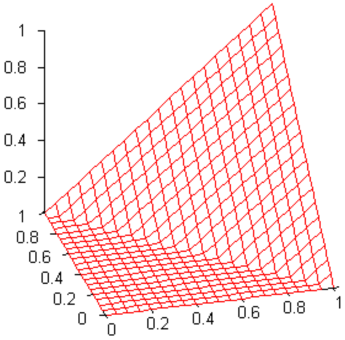


Figure 8: **product** T-norm

The Most Common T-norms

$$\mathbf{T_{Luk}}(a,b) = max\{0,\ a+b-1\}$$



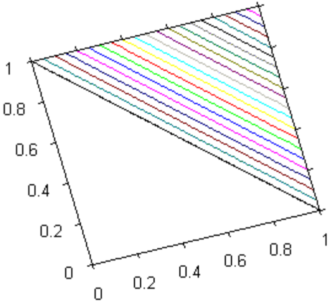


Figure 9: **Łukasiewics** T-norm

Fuzzy Control

- The wider application of the fuzzy logic [3]
- Easier to mechanize tasks that are already successfully performed by humans

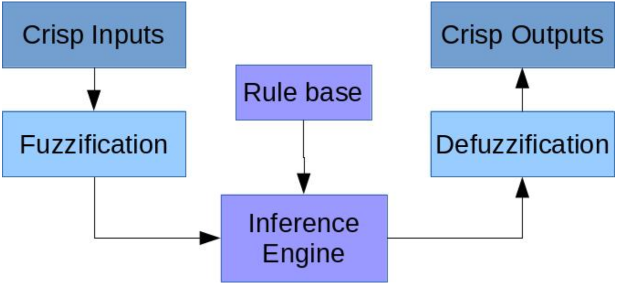


Figure 10: Block diagram of a fuzzy control

Fuzzy Inference Engine

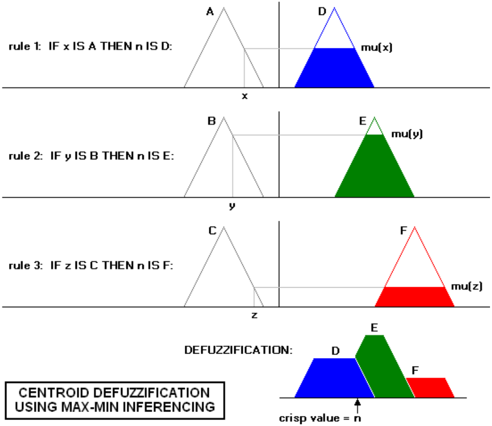


Figure 11: Process of a fuzzy control. The most used method for defuzzification is *center of gravity* (centroid).

Fuzzy Control Applications

- Camera autofocus by Canon
- Increased effectivity of Mutsushita vacuum robots
- Mitsubishi air conditioner with higher efficiency and lower sensors
- Handwriting recognition, elevator systems, self-balancing robots

The fuzzy control systems are commonly used [6] where there are not enough resources for highly advanced systems like **PID**[5] **controller**, **Artificial neural network** or **Genetic algorithm** [5].

[5] Proportional-integral-derivative

MATLAB Fuzzy Toolbox Introduction

- Provides a complete set of functions to design an implement various fuzzy logic processes [7]
- Major fuzzy logic operation-fuzzification, defuzzification, and the fuzzy inference
- Can be implemented using the Graphical User Interface (GUI)

MATLAB Fuzzy Toolbox

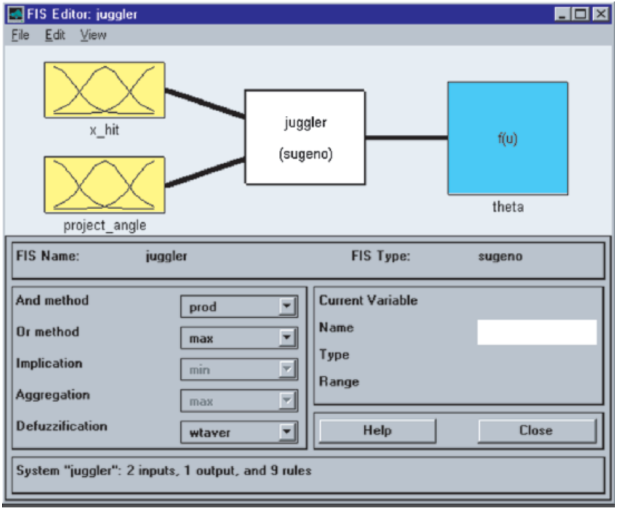
Features:

- It provides tools to create and edit fuzzy inference system (FIS).
- Allows integrating fuzzy systems into simulation with Simulink.
- It is possible to create stand-alone C programs that call on fuzzy systems

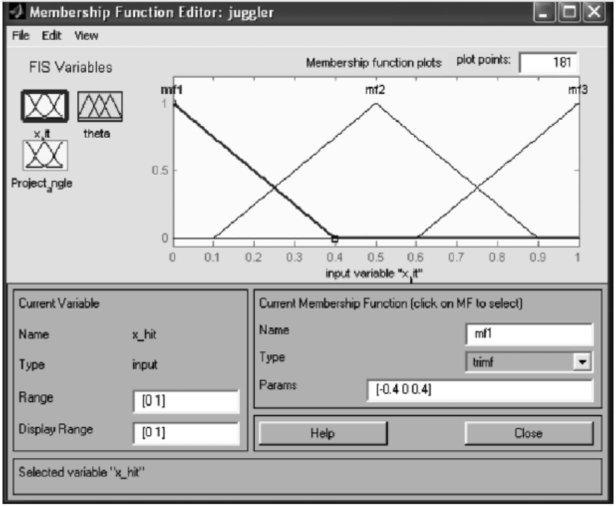
MATLAB Fuzzy Toolbox Tool Categories:

- Command line functions
- Graphical or interactive tools
- Simulink blocks

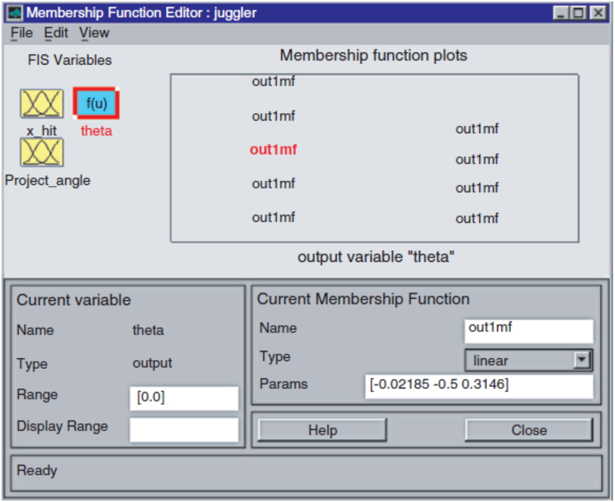
MATLAB Fuzzy Toolbox I



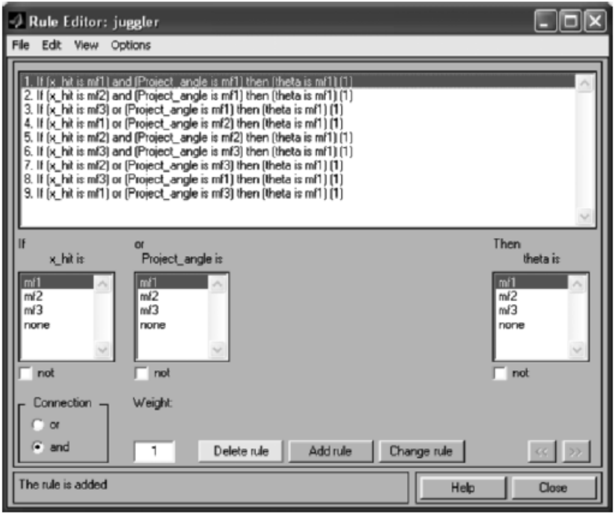
MATLAB Fuzzy Toolbox II



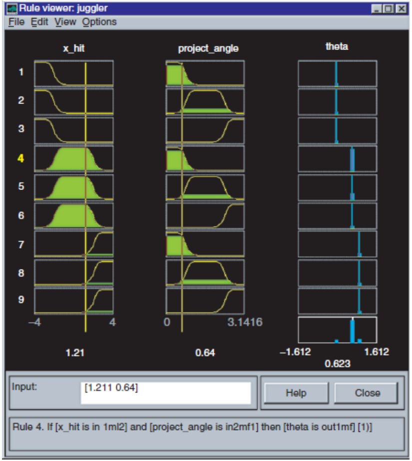
MATLAB Fuzzy Toolbox III



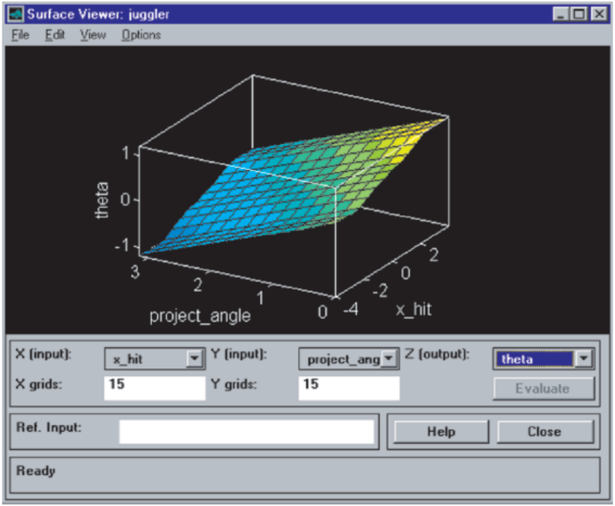
MATLAB Fuzzy Toolbox IV



MATLAB Fuzzy Toolbox V



MATLAB Fuzzy Toolbox VI



Is Fuzzy Logic a Viable Option?

The widespread use, amount of knowledge accumulated and countless tools and literature available proof it as **yes**.

References I

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[7] S.N. Sivanandam, S. Sumathi, and S.N. Deepa. *Introduction to Fuzzy Logic using MATLAB*. Springer Berlin Heidelberg, 2006. ISBN: 9783540357810.

Questions?

Thank you • ¡Gracias! • Ďakujeme