Dynamics in Electrical Systems

Jakub Hanak, Peter Babic
Dept. of Computers and Informatics, FEI TU of Kosice
Slovak Republic
jakub.hanak2@gmail.com, babicpet@gmail.com

***Abstract*—The abstract goes here.**

***Index Terms*—differential, dynamics, electrical, equation, modeling, ordinary, system**

I. Introduction

THIS this paper is intended to sum up the research done in order to understand the Dynamics in electrical systems and their underlying differential equations.

June 03, 2015

II. Dynamical System

adfsdf

III. Differential Equations

sadf

A. Slope field

In *mathematics*, a **slope field** (or **direction field**) is a graphical representation of the solutions of a first-order differential equation. It is useful because it can be created without solving the differential equation analytically. The representation may be used to qualitatively visualize solutions, or to numerically approximate them [3].

IV. Limit Cycle

A **limit cycle** is an isolated closed trajectory. *Isolated* means that neighboring trajectories are not closed - they spiral either towards or away from the limit cycle.

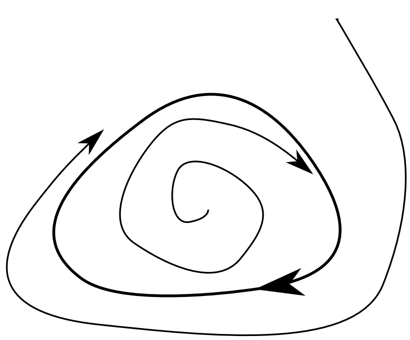

Fig. 1. Stable limit cycle. Trajectories spiral towards it.

If all neighboring trajectories approach the limit cycle, we say the limit cycle is **stable** or *attracting*, as shown on fig. 1. Otherwise the limit cycle is **unstable**, or in exceptional cases, **half-stable**. Stable limit cycles are very important scientifically they model systems that exhibit self-sustained oscillations. In other words, these systems oscillate even in the absence of external periodic forcing.

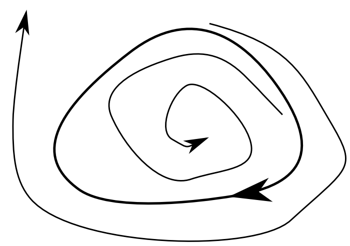

Fig. 2. Unstable limit cycle. Trajectories spiral away from it.

Of the countless examples that could be given, we mention only a few: the beating of a heart; the periodic ring of a pace maker neuron; daily rhythms in human body temperature and hormone secretion; chemical reactions that oscillate spontaneously; and dangerous self-excited vibrations in bridges and airplane wings. In each case, there is a standard oscillation of some preferred period, waveform, and amplitude. If the system is perturbed slightly, it always returns to the standard cycle. Limit cycles are inherently nonlinear phenomena; they cant occur in linear systems [4].

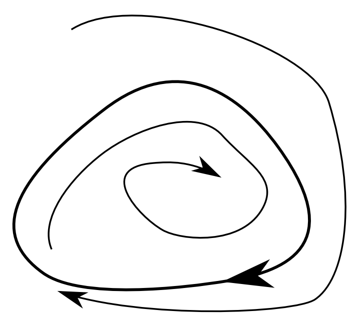

Fig. 3. Half-stable (or semi-stable) limit cycle. Attract trajectories from one side and repel them from other side.

A. Liénard Equation

A non-linear second-order ordinary differential equation

$$y'' + f(x)x' + x = 0 \tag{1}$$

This equation describes the dynamics of a system with one degree of freedom in the presence of a linear restoring force and non-linear damping. If the function f has the property

B. Van der Pol Oscillator

The limit cycle property translated to real life application means oscillations. Oscillations are important part of electronics [1]. One of the most well-known oscillator model in dynamics is **Van der Pol oscillator**, described by differential equation

$$y'' - \mu\left(1 - y^2\right)y' + y = 0 \tag{2}$$

where y is the dynamical variable and $\mu > 0$ is a parameter. If $\mu = 0$, then the equation reduces to the equation of simple harmonic motion

$$y'' + y = 0$$

The Van der Pol equation (2) arises in the study of circuits containing vacuum tubes and is derived from earlier, Rayleigh equation [2], known also as Rayleigh-Plesset equation - an ordinary differential equation explaining the dynamics of a spherical bubble in an infinite body of liquid.

Van der Pol oscillator is **self-sustainable**, **relaxation** oscillator. Self-sustainability in this context means, that the energy is fed into small oscillations and removed from large oscillations. Relaxation means, that the energy is gradually accumulating over time and then quickly released (relaxed). In electronics jargon, the relaxation oscillator is also called a *free-running* oscillator. As already explained, it does not require neither one (monostable), nor two (bistable) inputs for transitioning between states, it ”runs” by itself, thus free-running.

V. Conclusion

The conclusion goes here.

Appendix A
Proof of the First Zonklar Equation

Appendix one text goes here.

Appendix B

Appendix two text goes here.

Acknowledgment

The authors would like to thank...

References

[1] G.C. K, gupta Sanjay, and garg Suresh. *Oscillations and Waves*. Prentice-Hall Of India Pvt. Limited. ISBN: 9788120339217.

[2] P.J. Nahin. *The Science of Radio.: With Matlab and Electronics Workbench Demonstration, 2nd edition*. Online files. Springer New York, 2001, p. 96. ISBN: 9780387951508.

[3] S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Advanced book program. Westview Press, 1994. ISBN: 9780738204536.

[4] S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Studies in nonlinearity. Westview Press, 2008. ISBN: 9780786723959.