Dynamics in Electrical Systems

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***Abstract*—The abstract goes here.**

***Index Terms*—differential, dynamics, electrical, equation, modeling, ordinary, system**

I. Introduction

THIS this paper is intended to sum up the research done in order to understand the Dynamics in electrical systems and their underlying differential equations.

June 03, 2015

II. Dynamical System

Dynamical systems are mathematical objects used to model physical phenomena whose state (or instantaneous description) changes over time. These models are used in financial and economic forecasting, environmental modeling, medical diagnosis, industrial equipment diagnosis, and a host of other applications.

For the most part, applications fall into three broad categories: predictive (also referred to as generative), in which the objective is to predict future states of the system from observations of the past and present states of the system, diagnostic, in which the objective is to infer what possible past states of the system might have led to the present state of the system (or observations leading up to the present state), and, finally, applications in which the objective is neither to predict the future nor explain the past but rather to provide a theory for the physical phenomena. These three categories correspond roughly to the need to predict, explain, and understand physical phenomena.

III. Differential Equations

A **differential equation** is any equation which contains derivatives, either ordinary derivatives or partial derivatives.

A. Direction Field

Understanding **direction fields** (or **slope fields)** and what they tell us about a differential equation and its solution is important and can be introduced without any knowledge of how to solve a differential equation and so can be done here before we get into solving them. So, having some information about the solution to a differential equation without actually having the solution is a nice idea that needs some investigation.

Next, since we need a differential equation to work with this is a good section to show you that differential equations occur naturally in many cases and how we get them. Almost every physical situation that occurs in nature can be *described* with an appropriate differential equation.

The process of describing a physical situation with a differential equation is called **modeling**. We will be looking at modeling several times throughout this class.

The direction fields are important because they can provide a *sketch of solution*, if exist, and a *long term behavior* - most of the time we are interested in general picture about what is happening, as the time passes.

IV. Limit Cycle

A **limit cycle** is an isolated closed trajectory. *Isolated* means that neighboring trajectories are not closed - they spiral either towards or away from the limit cycle. The particle on the limit cycle, appears after one period on the exact same spot.

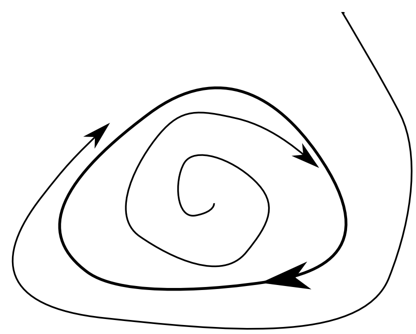

Fig. 1. Stable limit cycle. Trajectories spiral towards it.

If all neighboring trajectories approach the limit cycle, we say the limit cycle is **stable** or *attracting*, as shown on fig. 1. Otherwise the limit cycle is **unstable**, or in exceptional cases, **half-stable**. Stable limit cycles are very important scientifically they model systems that exhibit self-sustained oscillations. In other words, these systems oscillate even in the absence of external periodic forcing.

Of the countless examples that could be given, we mention only a few: the beating of a heart; the periodic ring of a pace maker neuron; daily rhythms in human body temperature and hormone secretion; chemical reactions that oscillate spontaneously; and dangerous self-excited vibrations in bridges and airplane wings. In each case, there is a standard oscillation of some preferred period, waveform, and amplitude. Oscillations are important part of electronics [2], too.

If the system is perturbed slightly, it always returns to the standard cycle. Limit cycles are inherently nonlinear phenomena; they cant occur in linear systems [7] [6].

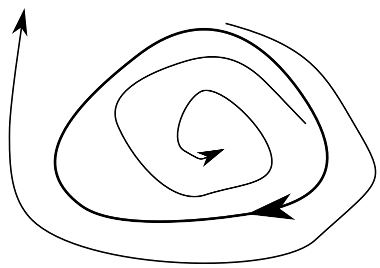

Fig. 2. Unstable limit cycle. Trajectories spiral away from it.

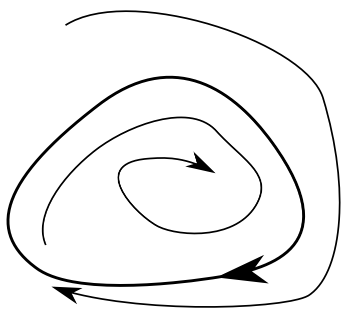

Fig. 3. Half-stable (or semi-stable) limit cycle. Attract trajectories from one side and repel them from other side.

A. Damping

Mentioning damping is important mainly because, in a real world, oscillations eventually stop, due to Newton's law of Thermodynamics (the frictional force). In electronics, there is no ideal oscillator, too - small amount of energy is lost every cycle, due to electric resistance.

Generally, the damping is linear either linear or non-linear. As a rule of thumb, the linear one is easily modeled mathematically, obeying known rules, while the non-linear one is not [1]. There are some use cases, where non-linear damping is advantageous, but the research is still ongoing about this topic.

B. Liénard Equation

A non-linear second-order ordinary differential equation

$$y'' + f(x)x' + x = 0 \tag{1}$$

This equation describes the dynamics of a system with one degree of freedom in the presence of a linear restoring force and non-linear damping. The function f has properties

$$f(x) < 0 \quad for\, small\, |x|$$
$$f(x) > 0 \quad for\, large\, |x|$$

that is, if for small amplitudes the system absorbs energy and for large amplitudes dissipation occurs, then in the system one can expect self-exciting oscillations.

Liénard equation was intensely studied as it can be used to model oscillating circuits. Under certain additional assumptions Liénard's theorem guarantees the uniqueness and existence of a limit cycle for such a system.

C. Van der Pol Oscillator

One of the most well-known oscillator model in dynamics is **Van der Pol oscillator**, which is a special case of Liénard's equation (1) and is described by a differential equation

$$y'' - \mu\left(1 - y^2\right) y' + y = 0 \tag{2}$$

where y is the dynamical variable and $\mu > 0$ is a parameter. If $\mu = 0$, then the equation reduces to the equation of simple harmonic motion

$$y'' + y = 0$$

The μ parameter determines the shape of the limit cycle. As it approaches 0, it gets the shape of a circle. On the other hand, increasing the paramter, involves sharpening of the curves.

The Van der Pol equation (2) arises in the study of circuits containing vacuum tubes (triode) and is derived from earlier, Rayleigh equation [4], known also as Rayleigh-Plesset equation - an ordinary differential equation explaining the dynamics of a spherical bubble in an infinite body of liquid.

Van der Pol oscillator is **self-sustainable**, **relaxation** oscillator. Self-sustainability in this context means, that the energy is fed into small oscillations and removed from large oscillations. Relaxation means, that the energy is gradually accumulating over time and then quickly released (relaxed). In electronics jargon, the relaxation oscillator is also called a *free-running* oscillator. As already explained, it does not require neither one (monostable), nor two (bistable) inputs for transitioning between states, it "runs" by itself, thus free-running.

D. Van der Pol's Equation Limit Cycle

Liénard's theorem can be used to prove that the system described by Van der Pol equation (2) has a limit cycle [5]. If we want to visualize it, the one-dimensional form of equation must be first *transformed* to the two-dimensional form. Applying the Liénard transformation

$$y = x - \frac{x^3}{3} - \frac{\dot{x}}{\mu}$$

where dot indicates the time derivative, the system can be written in it's two-dimensional form [3]:

$$\dot{x} = \mu\left(x - \frac{1}{3}x^3 - y\right)$$
$$\dot{y} = \frac{1}{\mu}x$$

However, this form is not well-known. Far common form uses the transformation $y = \dot{x}$, that yields

$$\dot{x} = y$$
$$\dot{y} = \mu\left(1 - x^2\right) y - x$$

which can be plotted onto direction field, as shown on fig. 4. It is possible to see the stable limit cycle as well as trajectories from both sides attracted towards it.

The Van der Pol oscillator can be forced too, however, this work does not aim to investigate further in this direction.

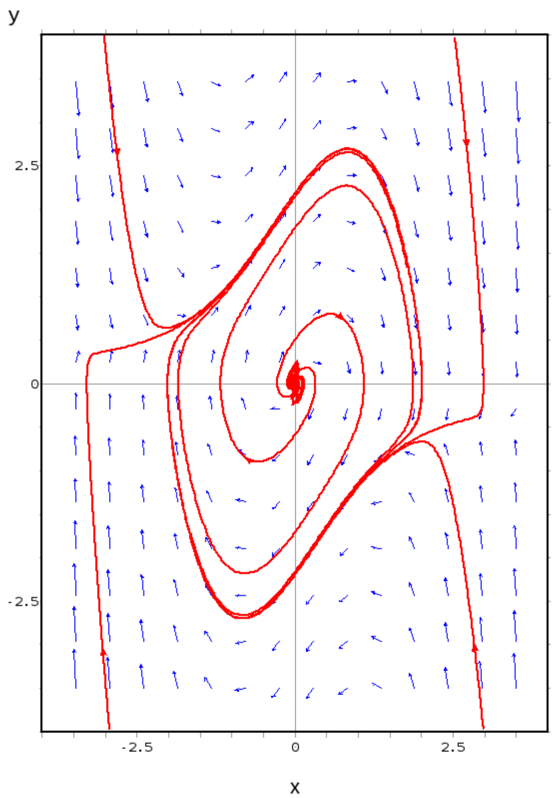

Fig. 4. Phase portrait of the unforced Van der Pol oscillator, showing a limit cycle and the direction field Parameter $\mu = 1$. The wxMaxima computing software was used for this purpose.

V. Conclusion

The conclusion goes here.

Appendix A
Proof of the First Zonklar Equation

Appendix one text goes here.

Appendix B

Appendix two text goes here.

Acknowledgment

The authors would like to thank...

References

[1] Institute for Computer Applications in Science et al. *Estimation of Nonlinear Damping in Second Order Distributed Parameter Systems*. 1989.

[2] S. Garg, C.K. Gosh, and S. Gupta. *Oscillations and Waves*. Prentice-Hall Of India Pvt. Limited. ISBN: 9788120339217.

[3] D. Kaplan and L. Glass. *Understanding Nonlinear Dynamics*. Textbooks in Mathematical Sciences. Springer New York, 2012, pp. 240–244. ISBN: 9781461208235.

[4] P.J. Nahin. *The Science of Radio.: With Matlab and Electronics Workbench Demonstration, 2nd edition*. Online files. Springer New York, 2001, p. 96. ISBN: 9780387951508.

[5] S. Sternberg. *Dynamical Systems*. Dover Publications, 2014, p. 211. ISBN: 9780486135144.

[6] S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Advanced book program. Westview Press, 1994. ISBN: 9780738204536.

[7] S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Studies in nonlinearity. Westview Press, 2008. ISBN: 9780786723959.